Can you jam just one particle?

JAMMING THE RANDOM LORENTZ GAS [1]

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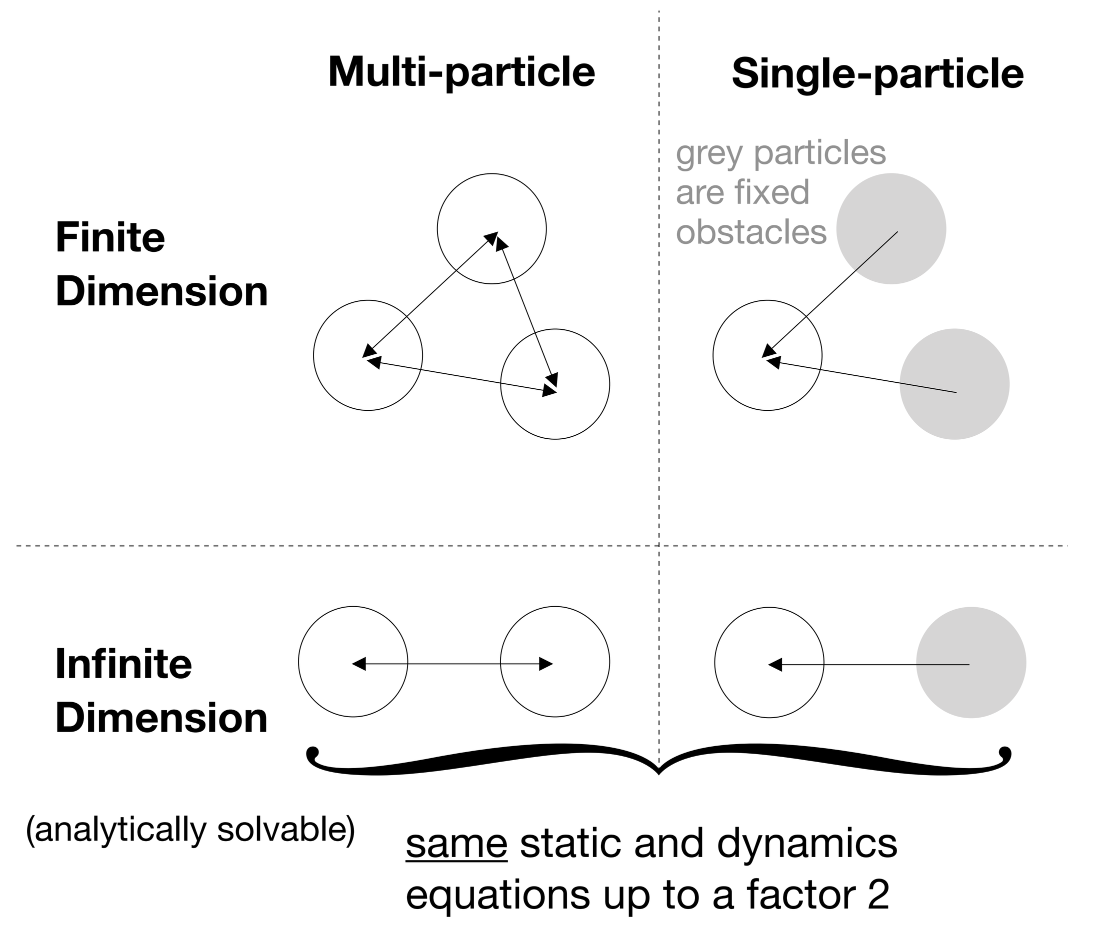
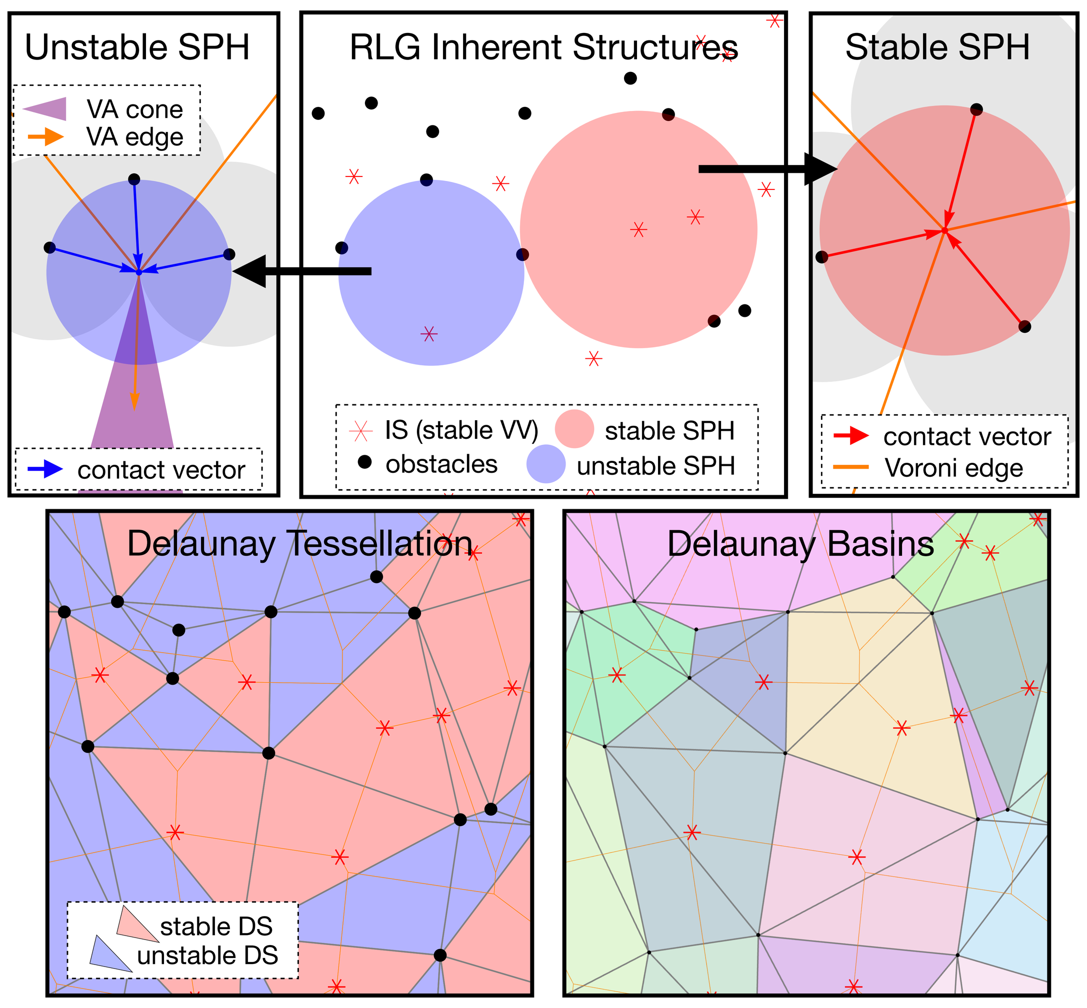
RANDOM LORENTZ GAS & VORONOI/DELAUNAY TESSELLATION

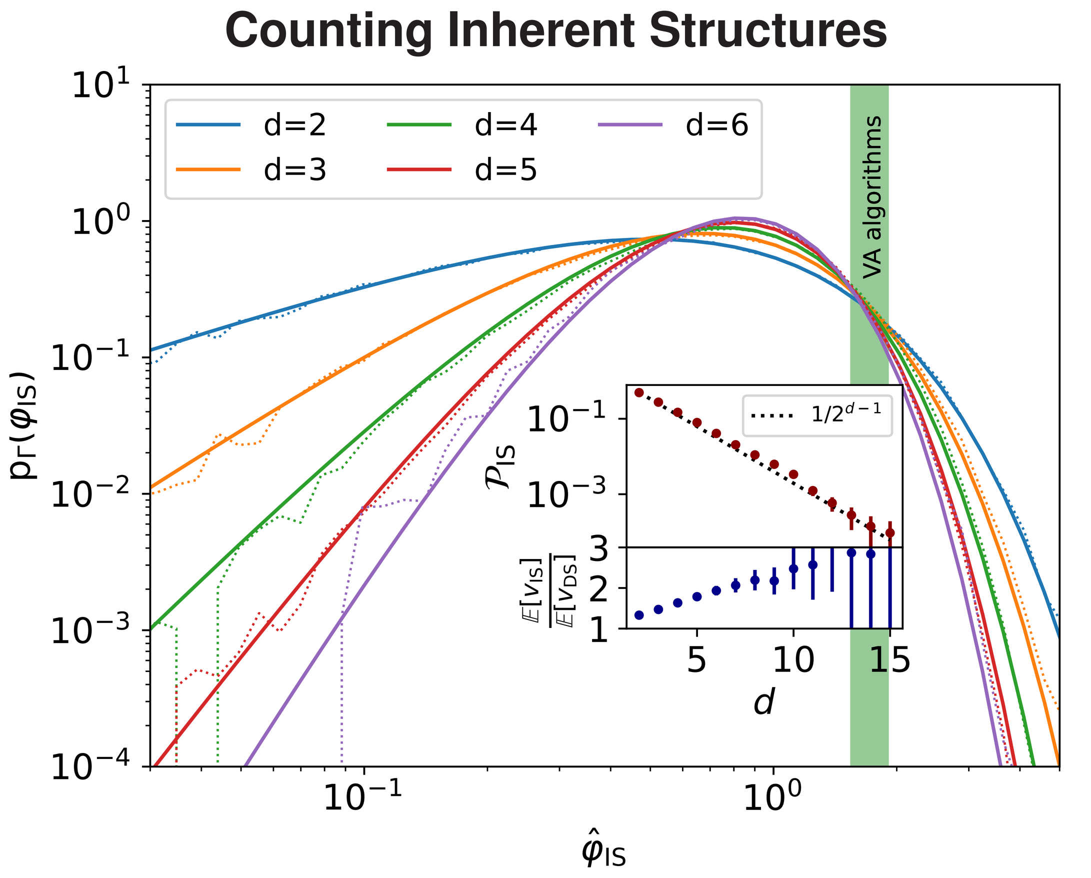


The **random Lorentz gas (RLG)** models a single spherical tracer evolving through randomly distributed, fixed point obstacles. A general **compression protocol** involves increasing the tracer radius until it is no longer possible to do so locally. The overdamped limit of these protocols defines a class of **volume ascent (VA)** algorithms. When the tracer is in contact with $d+1$ obstacles, it is **stable** (red circle) if these obstacles are not cohemispheric [4]; otherwise it is **unstable** (blue circle) and growth directions remain, thus defining a VA cone (purple). This process maps to **Voronoi/Delaunay tessellations**: stable/unstable configurations lie at Voronoi vertices (Delaunay circumcenters), and a vertex is stable if it lies within its corresponding **Delaunay simplex** (DS, red triangles). In $d=2$, about half of the triangles are stable, but in higher dimensions, only an exponentially small fraction (in d) of these Delaunay simplexes is stable. Maximal VA trajectories (VA-max)—the non-smooth analog of gradient descent—lead to basins formed by assemblies of unstable Delaunay simplexes clustered around a stable one, thus defining **Delaunay basins** (see colored basins).



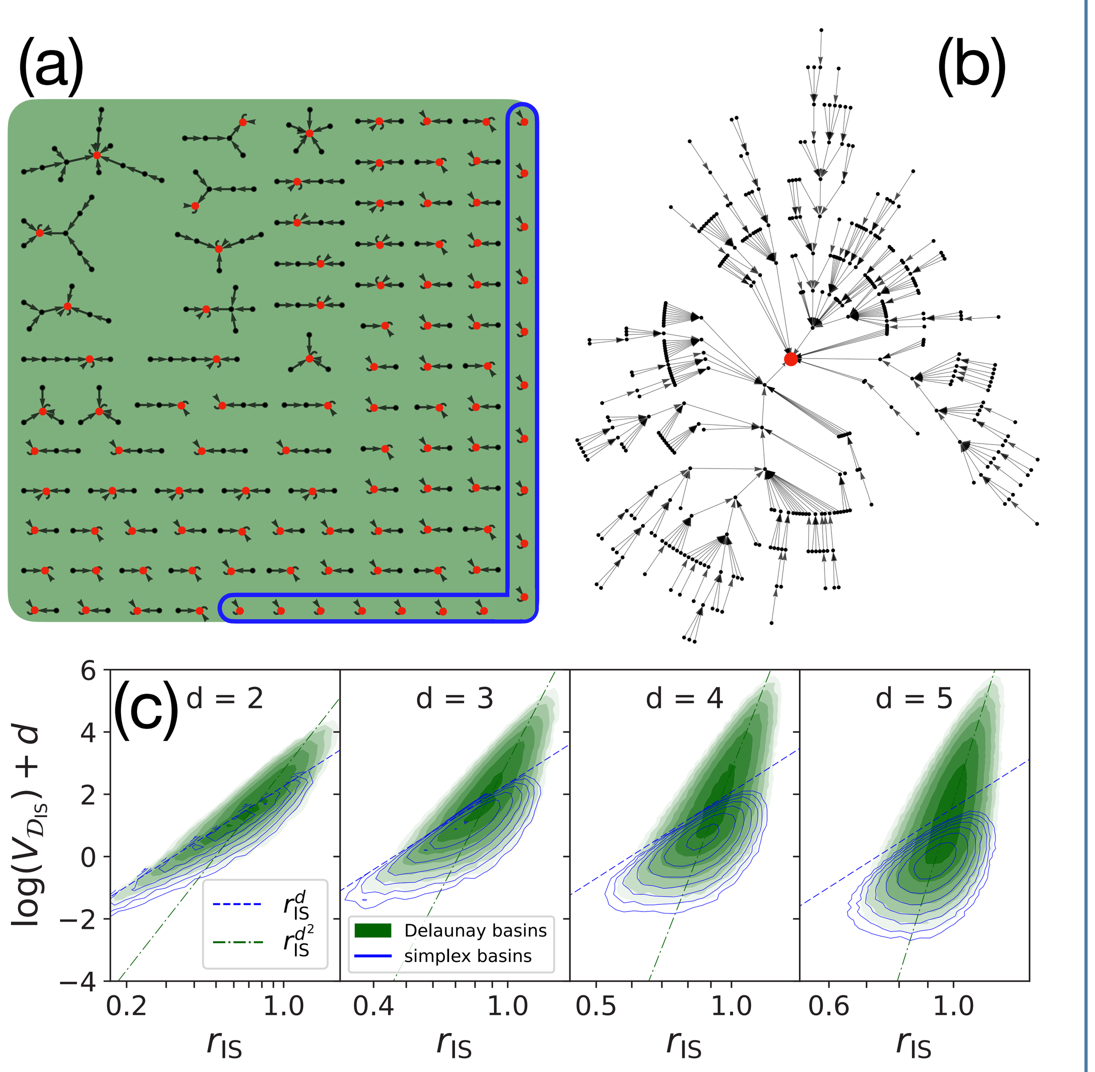
OPTIMIZATION IN THE NON-SMOOTH RLG-HS LANDSCAPE

Counting Inherent Structures



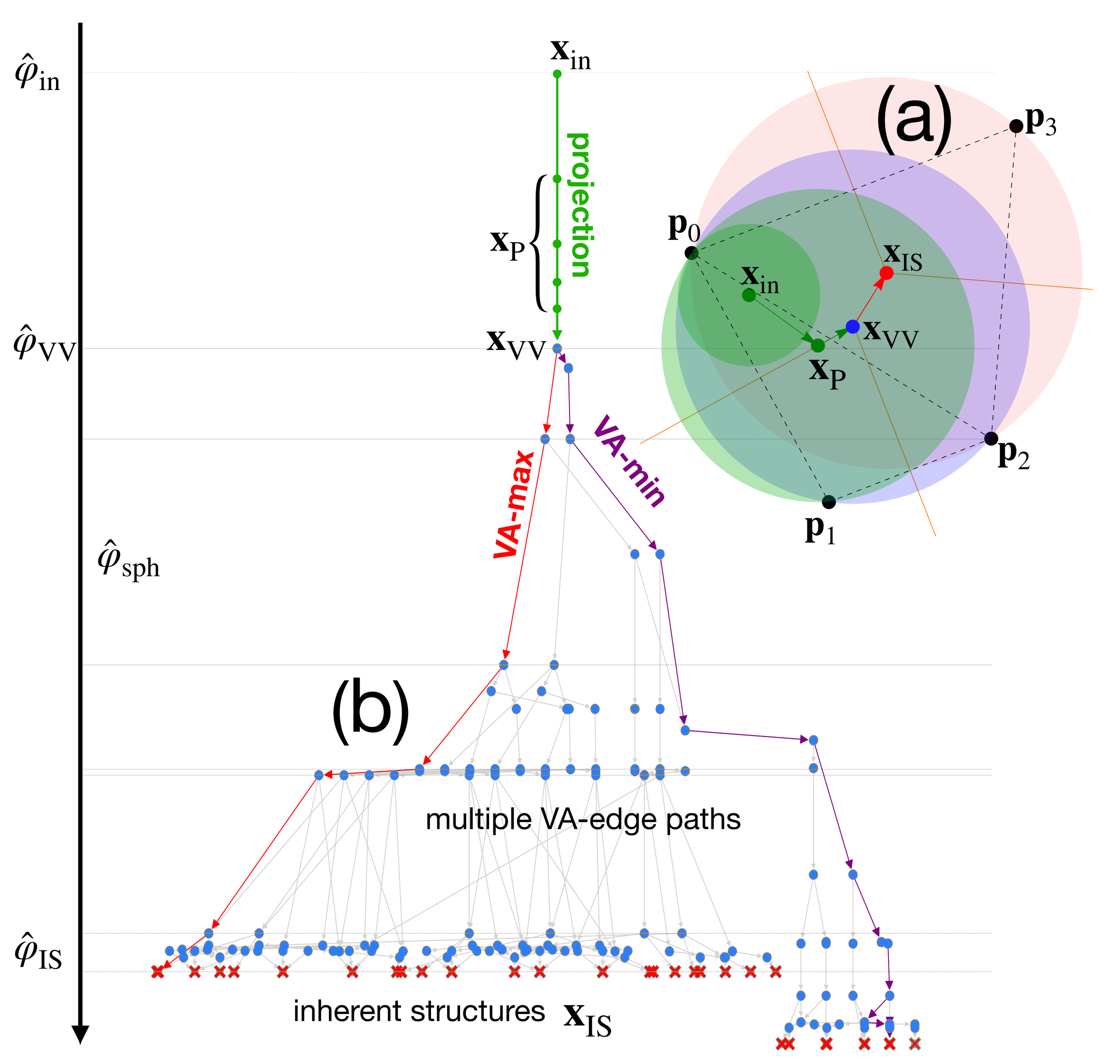
In $d=2-6$, the **packing fraction distribution** of inherent structures (ISs) follows a **Gamma law** (full lines), peaking at $(d-1)/d$ with *power-law* left and *exponential* right tails (dotted lines are numerical results). The *mean packing fraction* is 1 in all d, but **VA algorithms** typically reach higher values. Each IS is the **circumcenter of a stable Delaunay simplex**. In high d, stable DSs (aka ISs) are **exponentially rare** — both in number and in volume.

Shape of Delaunay Basins



Delaunay basins under VA-max form *tree-like graphs* **(a)(b)**, rooted at stable simplexes and composed of hierarchically nested unstable ones. Their size grows rapidly with dimension, exhibiting **fractal scaling** as volume $\sim r^{d^2}$ **(c)**. **Greedy algorithms** like VA-max favor larger basins, leading to a sampling bias toward **denser ISs**.

Volume Ascent Algorithms



Despite the **rarity of stable DS**, VA algorithms can still reach them efficiently by monotonically increasing the tracer radius $r_{\rm sph}$ **(a)**. These schemes evolve a *non-smooth, kinked landscape*, for which standard optimization fails. **Each unstable Delaunay simplex** is associated with a Voronoi vertex (VV) and a **cone of ascent directions**.

- In $d=2$, each VV has a single VA edge.
- In $d>2$, the number of VA edges varies from 1 to $d-1$.

Due to the exponential scarcity of stable DSs, VA algorithms must traverse many unstable VVs to reach an inherent structure (IS). The resulting **ascent graph (b)** is characterized by:

- *Multifurcation*: many branching paths (common at high d)
- *Coalescence*: rare merging of paths (vanishes as d increases)

We analyze two extreme strategies:

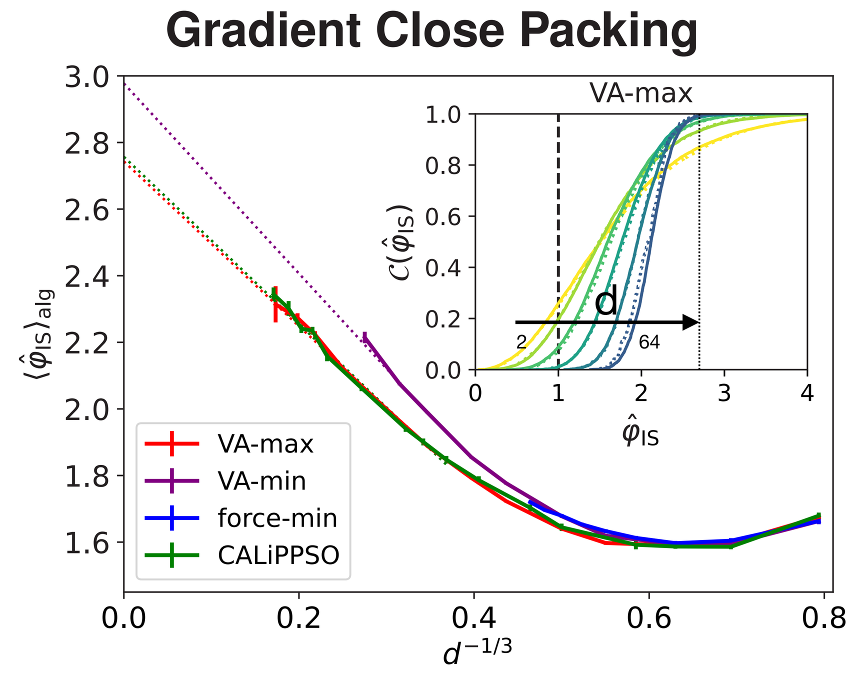
- **VA-max** (greedy): takes the steepest ascent at each VV
- **VA-min** (reluctant): always takes the shallowest available path

More reluctant algorithms (like VA-min) typically reach higher $\hat{\varphi}_{\rm IS}$ over longer paths. VA-max finds lower-density ISs over shorter paths.

In high dimensions, the VA landscape is rough, and algorithmic choice crucially affects which ISs are found and how efficiently.

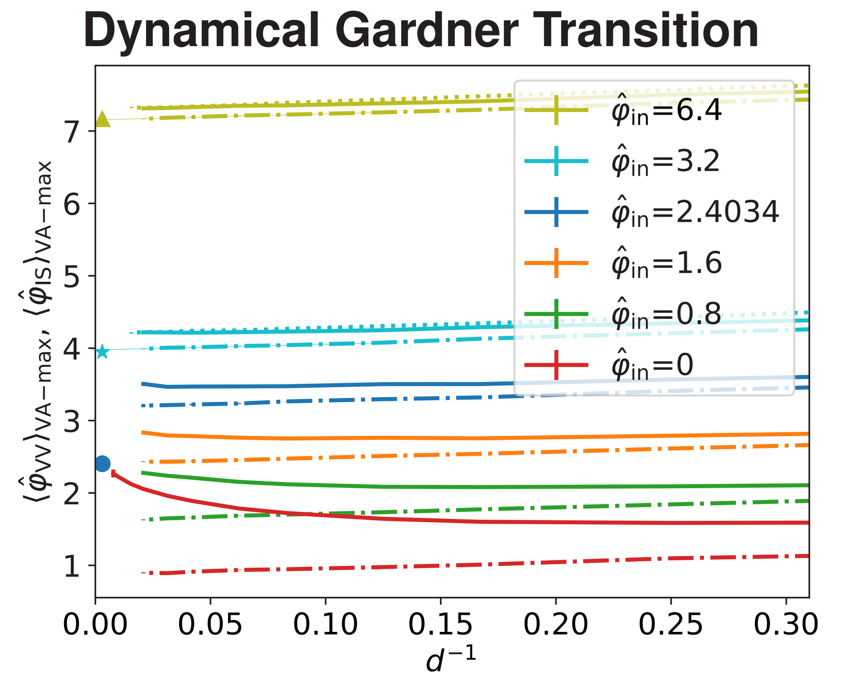
GRADIENT CLOSE PACKING, GARDNER and JAMMING

Gradient Close Packing



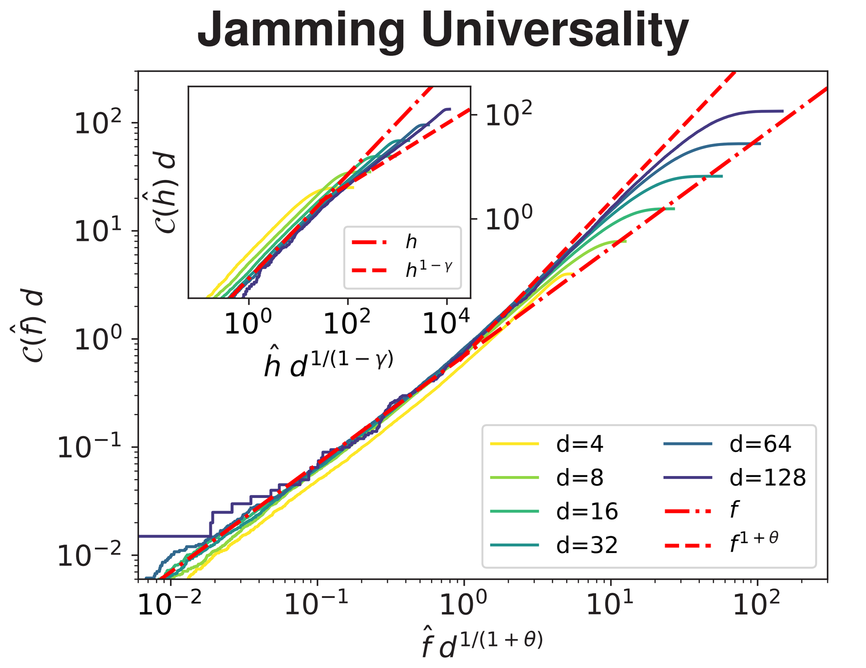
VA-max and CALiPPSO [5] reach similar jamming densities, scaling as $d^{-1/3}$ and converging to $\hat{\varphi}_{\rm J0}\approx 2.73$ as $d\to\infty$. In contrast, VA-min and force-min achieve denser packings (≈ 2.94). The distribution of $\hat{\varphi}_{\rm IS}$ sharpens with increasing d.

Dynamical Gardner Transition



For VA-max (full lines), the final jamming density $\langle\hat{\varphi}_{\rm IS}\rangle$ and intermediate value at the first VV (dashed-dotted) both grow with initial density $\hat{\varphi}_{\rm in}$. At high $\hat{\varphi}_{\rm in}$, well above the dynamical transition $\hat{\varphi}_{\rm d}$, extrapolated VV values align with the static $d\to\infty$ Gardner transition (star,triangle); for lower $\hat{\varphi}_{\rm in}$ they do not.

Jamming Universality



Rescaled force distributions from VA-max show good collapse and a crossover to a power-law tail with the mean-field exponent $\theta\approx 0.423$ [3]. Gap distributions do not display a similar power-law, consistent with larger finite-dimensional corrections.

PERSPECTIVES

We investigated the **random Lorentz gas** and a class of **volume ascent (VA)** algorithms to study real-space jamming. In high dimensions, the landscape is dominated by **unstable regions**, leading to **rough, hierarchical, and fractal-like basins of attraction**. The **greedy VA-max** algorithm efficiently estimates the asymptotic jamming density $\varphi_{\rm J0}$ and reveals a **dynamical Gardner transition** consistent with mean-field theory. As $d\to\infty$, jammed configurations become **algorithm-independent**, indicating a **geometric universality** in the jamming process. VA schemes are **potentially extendable to multi-particle systems** and may provide a scalable route to estimating jamming thresholds. More broadly, they offer a **computationally efficient approach** to high-dimensional **robust optimization** problems. In particular, **VA-min** explores configuration space thoroughly with modest computational cost, avoiding expensive tessellations.

REFERENCES

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